A Method for a Faithful Reconstruction of an Off-Axis Type Ultrasound Holography

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Abstract

The method for a fully faithful reconstruction of an acoustic holography proposed by J. Stone is generalized such that it can be applied to an off-axis type holography as well as an in-line type holography with which he dealt.

Inhalt

Eine Methode zur verzeichnungsfreien Rekonstruktion von Ultraschall-Hologrammen mit off-axis-Typ. Eine Methode von J. Stone zur verzeichnungsfreien Rekonstruktion von akustischen Hologrammen wird für off-axis Holographie verallgemeinert.

One of the most important problems in acoustic holography lies in the presentation of the information contained in a hologram. The method used in the reconstruction from optical holograms can be used, but, due to the wavelength change, a primary longitudinal distortion is introduced. A faithful reconstruction free of this distortion can generally be obtained when the hologram is reduced inversely as the ratio of recording to reconstructing wavelength [1], [2], but this in turn reduces the image too much to be viewed normally [3], [4]. Several techniques without the hologram reduction have been proposed to reduce this distortion by means of "fractionated" hologram method [5], [6], binocular visual system using a pair of prisms [7] and other method [8]. It seems, however, that all these methods will be of little value in reducing the distortion by more than a factor of about 20 because of diffraction effects or technical difficulties. An interesting technique which made it possible to reduce that factor more large has also been reported [9], but it can only be applied to an in-line holography. A somewhat similar technique proposed by us [10] is more simple to use. The purpose of this letter is to give a generalized method of those in references [9] and [10]; that is, the method proposed here is applicable to an off-axis holography as well as in-line holography.

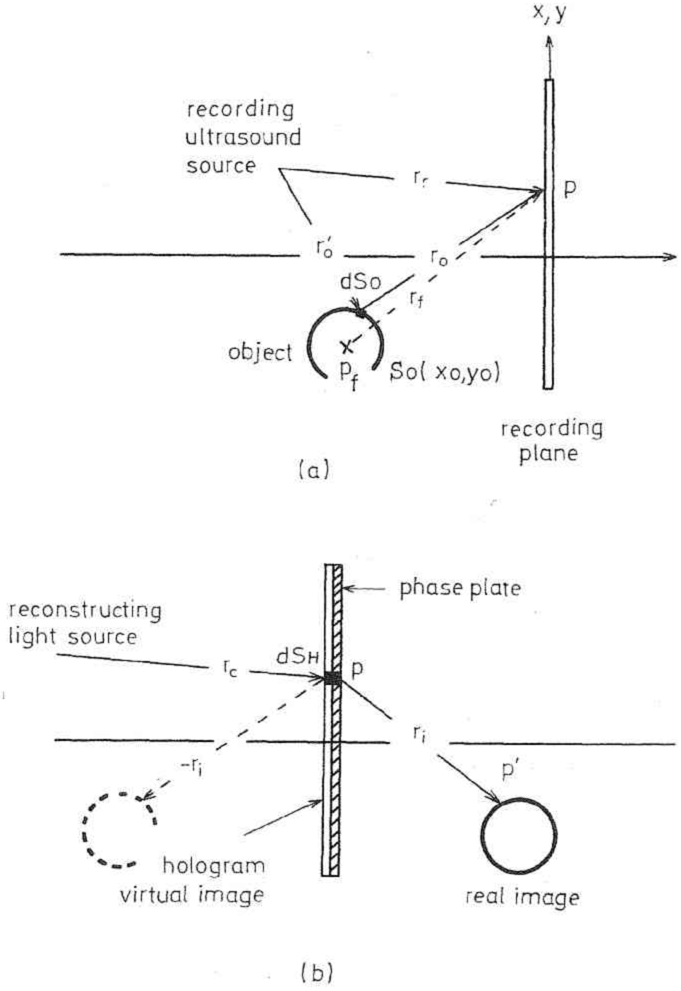

Fig. 1. Holographic geometry, (a) recording geometry, (b) reconstructing geometry.

Fig. 1 shows the holographic geometry. The hologram is recorded by exposure of the photographic plate with simultaneous illumination from both the object and the reference source of wavelength λ_0. The complex amplitude of the radiation falling on the recording plane at P is then

$$U(x, y) = \frac{i}{\lambda_0} \int \frac{A_0}{r_0} \cdot \exp\left[i\,(2\pi/\lambda_0)\,(r_0' + r_0)\right] \cdot S_0(x_0, y_0)\, dS_0$$

$$+\frac{A_r}{r_r}\cdot\exp\left[i\left(2\pi/\lambda_0\right)r_r\right] \tag{1}$$

where S_0 represents a scattering coefficient of the object and r_0, r_0' and r_r are the distances. The integration is taken over the entire surface of the object. The exposed plate is then processed so that it has a transmittance proportional to the intensity distribution of the interference pattern. The plate i.e., hologram has then an amplitude transmittance $I(x, y)$ given by

$$I(x, y) = |U(x, y)|^2. \tag{2}$$

The reconstruction can be accomplished by the same method as optical holography with a visible coherent light source of wavelength λ_c, as shown in Fig. 1(b). In this case, however, a phase plate of complex amplitude transmittance $\exp[i\Phi(x, y)]$ is placed just behind the hologram to eliminate the distortion due to the wavelength change. The resultant wave at a field point P' is then

$$U_i'(r_i) = \frac{i}{\lambda_c}\int\frac{A_c}{r_c}\cdot\exp\left[i\left(2\pi/\lambda_c\right)r_c\right]\cdot I(x, y)\cdot\exp\left[i\Phi(x, y)\right]$$

$$\times\frac{1}{r_i}\cdot\exp\left[i\left(2\pi/\lambda_c\right)r_i\right]dS_H, \tag{3}$$

where the integration is taken over the entire hologram plane. If the phase plate is removed and the reconstruction is done with the original ultrasound source, the resulting wave at that point is

$$U_i(r_i) = \frac{i}{\lambda_0}\int\frac{A_0'}{r_c}\cdot\exp\left[i\left(2\pi/\lambda_0\right)r_c\right]\cdot I(x, y)\cdot\frac{1}{r_i}$$

$$\times\exp\left[i\left(2\pi/\lambda_0\right)r_i\right]dS_H, \tag{4}$$

and a perfect reconstruction without the distortion may be obtained. The $1/r$ amplitude dependence is assumed to be constant hereafter, that is, each radiation from a point source has a constant intensity across the planes concerned. The assumption may naturally be satisfied in the long wavelength holography considered here. The phase requirement of the plate for a fully faithful reconstruction can, therefore, be obtained by equating $U_i'(r_i)$ of (3) and $U_i(r_i)$ of (4)

$$\Phi = -2\pi\left[\left(1/\lambda_c - 1/\lambda_0\right)\left(r_c + r_i\right)\right]. \tag{5$'$}$$

It becomes

$$\Phi = -2\pi\,[(1/\lambda_c - 1/\lambda_0)\,(r_c + r_0)] \tag{5}$$

when applied to a conjugate image i.e., real image since, in this case, $r_i = r_0$. Such a phase plate can be constructed precisely by recording a hologram of the original object S_0 at a wavelength λ

$$1/\lambda = 1/\lambda_c - 1/\lambda_0 \tag{6}$$

with the same holographic geometry as the original one. But the recording source should be placed at the original point or at the conjugate point according as the true image or the conjugate image is reconstructed. In the usual ultrasound holography, the ratio of reconstructing to recording wavelength $\mu\,(= \lambda_c/\lambda_0)$ is typically in the range of 10^{-3} to 10^{-4}. The wavelength requirement of (6) is then

$$\lambda \simeq \lambda_c\,(1 + \lambda_c/\lambda_0) = \lambda_c\,(1 + \mu) \simeq \lambda_c. \tag{7}$$

It will be clear that the method proposed in this letter is restricted to an optical holography in the recording step of the phase plate. Now consider a phase plate approximately capable of satisfying (5). Such a phase plate can be constructed simply by recording a hologram of a point P_f on the original object. The restriction in that step will then be removed and this method may easily be applied to the ultrasound holography. But the reconstruction problems will arise due to the phase plate. The phase of the plate Φ_f becomes

$$\Phi_f = -2\pi\,[(1/\lambda_c - 1/\lambda_0)\,(r_f + r_c)]\,\Big|_{\,r_c = -r_r} \tag{8}$$

and then the resulting waves at the field point P' is

$$U_i''\,(r_i) = \frac{i}{\lambda_c}\int \exp\,[i\,(2\pi/\lambda_0)\,r_c]\cdot I\,(x, y)\cdot \exp\,[i\,(2\pi/\lambda_0)\,r_i]$$

$$\times \exp\,[i\,2\pi\,(1/\lambda_c - 1/\lambda_0)\,(r_i - r_f)]\;dS_H. \tag{9}$$

It is easily seen, by comparing U_i'' with U_i which is the ideal wave, that U_i'' has an extraneous exponential term, $\exp\,[i\,2\pi\,(1/\lambda_c - 1/\lambda_0)\,(r_i - r_f)]$, in the wave. However it will only reduce the intensity of the image from the ideal value given by (4) and will not destroy the image quality since, as seen in (9), the information about the original object is contained in $I\,(x, y)$. The region where the ideal intensity is maintained may generally be restricted to

$$(r_i - r_f)/r_c,\ (r_i - r_f)/r_i \ll \lambda/\lambda_0. \tag{10}$$

To visualize the object everywhere, such a method as "ultrasonic stereoholography" proposed by B.D. Sollish and I. Glaser [11] may successfully be used; that is, a large number of elementary phase plate, each being a record of a different point on the object, are set in two dimensional array and are synthesized into a composite phase plate as shown in Fig. 2 and this plate is used for the reconstruction. The relation (10) will give a general standard for the region where the image is ideally reconstructed, but more correctly, it can only be answered by experiment or numerical analysis of eqs. (4) and (9). The analogous technique proposed in this letter can also be applied to such holography as X-ray or electron beam holography. In this case the

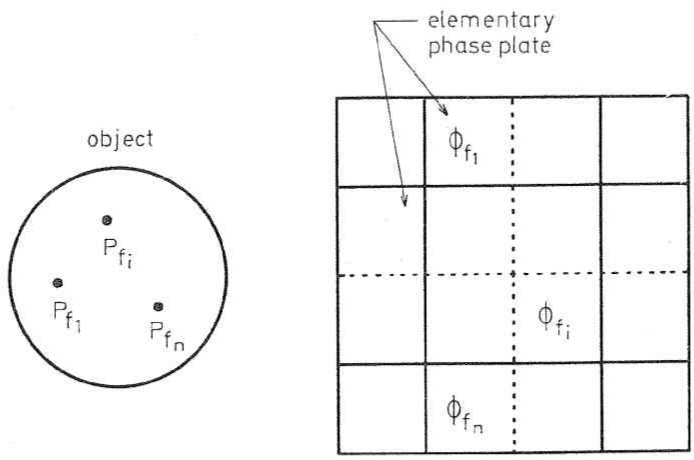

Fig. 2. Construction of a composite phase plate.

region to be ideally reconstructed is far wide in comparison with the case of ultrasound holography as follows

$$(r_i - r_f)/r_c,\ (r_i - r_f)/r_i \ll \lambda/\lambda_e \cong 1. \tag{11}$$

Where λ_e represents the wavelength of X-ray or electron beam.

When the above mentioned method is applied to an in-line holography where both the reference and recording waves are limited to a plane wave and are normal to the hologram plane, the phase requirement (5) is then reduced to

$$\Phi = -(2\pi/\lambda)\, r_i. \tag{12}$$

This is the case in references [9] and [10]. In this case, (12) can be satisfied to a good approximation simply by a Fresnel zone plate of focal length r_i, if the object subtends very small angle at the hologram and is located near the optical axis

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